LETTERS TO THE EDITOR

To the Editor:

This communication has been animated by the paper of Chu and Ng (January 1989, p. 148) entitled "Flow in Packed Tubes with a Small Tube to Particle Diameter Ratio." These authors investigated the impact of the tube to particle diameter ratio D/d on throughput and on head loss in packed tubes both theoretically and experimentally. The theoretical work of Chu and Ng was carried out by means of packed beds generated with a computer algorithm and is a continuation of previous efforts by Cohen and Metzner (1981) and by Nield (1983). The latter authors subdivided the cross-section of the packed tube in three regions, assuming that the relative spatial extent of each region depends on the diameter ratio D/d. Beside the theoretical analysis, Chu and Ng carried out measurements at low Reynolds numbers varying the diameter ratio D/d down to values of about 3. In this manner they completed previous investigations carried out at $D/d > 10$ and referred to Cohen and Metzner in some detail.

All the researchers mentioned above had the objective of finding out whether the diminution of the diameter ratio D/d gives rise to an increase or, on the contrary to a decrease of pressure drop, and to what extent it affects the pressure drop. They all recognized two different factors regarding the influence of D/d on pressure drop:

- The increased porosity of the bed in the vicinity of the wall results in an increased permeability and maldistribution of flow.
- The viscous friction at the surface area of the wall is not negligible in comparison to that of the particles for small values of D/d.

Inspite of the apparent simplicity of the problem and of the agreement on the physical background, diverging results have been presented till now. According to the analysis of Cohen and Metzner and to the experimental data for $D/d > 10$ collected by them, the pressure drop should increase monotonously with decreasing value of D/d. Chu and Ng also report a slight increase of pressure drop with decreasing diameter ratio for $D/d > 10$; they claim, however, that the functional dependence is inverted for $D/d > 10$. This behavior is supposed to be the characteristics of both the theoretical predictions and the experimental results. In order to explain their findings and to reconcile them with those of Cohen and Metzner, Chu and Ng assume that, the increased pressure drop at large values of D/d is due to wall friction, while the decreased head loss of small diameter ratios results from maldistribution of porosity and flow. Unfortunately, all previous calculations are quite complicated and hardly provide the means of distinguishing between the influence of wall friction and that of maldistribution on the final result.

In the following we make an effort to clarify the issue. We use a simple bisectional model proposed by Martin (1978) in order to describe maldistribution of porosity and flow in packed tubes. According to the model, the tube consists of a circular core ("1") having the porosity of the infinitely extended bed ($\psi_1 = \psi_\infty$) and of an annular region at the wall ("2") with the porosity $\psi_2(D/d) > \psi_\infty$. The thickness of the latter is assumed to be equal to a half particle diameter $d/2$. The model yields the average porosity of the bed $\bar{\psi}$ as a function of D/d. Furthermore, assuming that the pressure drop is the same in both sections of the tube and can be calculated from the Ergun Equation, the velocity ratios $u_{0,1}/\bar{u}_0$, $u_{0,2}/\bar{u}_0$ can be derived ($u_{0,i}$, $i = 1, 2$, superficial velocity in section "i," $\bar{u}_0$, average superficial velocity). Both velocity ratios depend on the Reynolds number $Re_0 = \bar{u}_0 d/\nu_f$ and, certainly, on the quotient D/d. The corresponding simple formulas are given in the original paper and need not be repeated here.

With the help of the bisectional model, we derive straightforward, very simple relationships for the influence of maldistribution on pressure drop. Doing so, we neglect the effect of wall friction. Further, we specify the reference situation implied when talking about "increased" or "decreased" pressure drop. The importance of this point is demonstrated by a simple analysis of the data of Chu and Ng. Actually, we show that the contradiction between these data and the findings of Cohen and Metzner for $D/d < 10$ is purely artificial and can be removed using the right reference situation. With the help of the reevaluated data of Chu and Ng, some remarks on the influence of wall friction are given.

Influence of maldistribution on pressure drop

With the help of the bisectional model of Martin, the pressure drop in the packed tube can be calculated accounting for the radial maldistribution of porosity and flow. We call this pressure drop ΔP_b ("b" for bisectional) and compare it with the head loss resulting for the same average velocity $\bar{u}_0$ in a homogeneous bed having the average porosity $\bar{\psi}$. The latter is denoted by ΔP_h ("h" for homogeneous). The quotient $\Delta P_b/\Delta P_h$ is nothing but a measure for the influence of maldistribution on pressure drop. If the latter is negligible, then $\Delta P_b/\Delta P_h \approx 1$ is obtained. Though $\Delta P_b/\Delta P_h$ can be calculated easily for any Reynolds number, we restrict ourselves in the following to very small or very large values of the latter. In these limiting cases we obtain

$$Re_0 \to 0 \sim \frac{\Delta P_b}{\Delta P_h} = \left(\frac{1-\psi_1}{1-\bar{\psi}}\right)^2 \left(\frac{\bar{\psi}}{\psi_1}\right)^3 \frac{u_{0,1}}{\bar{u}_0}, \quad (1a)$$

$$Re_0 \to \infty \sim \frac{\Delta P_b}{\Delta P_h} = \frac{1-\psi_1}{1-\bar{\psi}} \left(\frac{\bar{\psi}}{\psi_1}\right)^3 \left(\frac{u_{0,1}}{\bar{u}_0}\right)^2. \quad (1b)$$

Corresponding results are depicted in Figure 1 (solid line for $Re_0 \to 0$ and broken line for $Re_0 \to \infty$, with $\psi_1 = 0.38$). According to this figure, the maldistribution of porosity and flow always gives rise to a decrease of pressure drop ($\Delta P_b/\Delta P_h < 1$). This behavior is quite general and easy to understand. Actually, the combination of two resistances in parallel is always smaller than the corresponding average resistance. Furthermore, the influence of maldistribution on head loss appears to be quite small. In fact, $\Delta P_b/\Delta P_h$ is, in most cases, greater than 0.8, even for such small values of the diameter ratio ad $D/d = 5$.

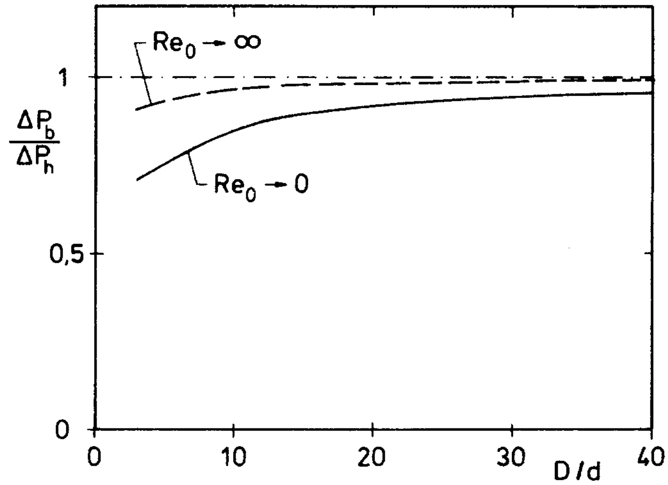

Figure 1. Influence of maldistribution on pressure drop.

On the reference pressure drop

In the foregoing we used the pressure drop ΔP_h of the homogeneous bed having the average porosity $\bar{\psi}$ as scaling factor. Completely different results are obtained if scaling is carried out with the pressure drop corresponding to the infinitely extended bed (denoted in the following by ΔP_∞). In order to quantify the effect, we calculate the quotient $\Delta P_h/\Delta P_\infty$ and get

$$Re_0 \to 0 \sim \frac{\Delta P_h}{\Delta P_\infty} = \left(\frac{1-\bar{\psi}}{1-\psi_\infty}\right)^2 \left(\frac{\psi_\infty}{\bar{\psi}}\right)^3, \quad (2a)$$

$$Re_0 \to \infty \sim \frac{\Delta P_h}{\Delta P_\infty} = \frac{1-\bar{\psi}}{1-\psi_\infty} \left(\frac{\psi_\infty}{\bar{\psi}}\right)^3. \quad (2b)$$

Corresponding results (with $\psi_\infty = 0.38$) are plotted in Figure 2. They show that pressure drop in narrow tubes is considerably lower than in the infinitely extended bed. The effect has nothing to do with flow maldistribution; it results from the change of average porosity with D/d. In fact, head loss calculations may not be carried out with the porosity ψ_∞. Instead, the actual average porosity of the packed tube $\bar{\psi}$ should be used. This simple rule has not been accounted for by Chu and Ng in the presentation of their results, giving rise to much confusion. The issue is discussed in the following in some detail.

Remarks on the data of Chu and Ng

At first we compare in Figure 3 the experimental data of Chu and Ng on the average porosity $\bar{\psi}$ (Figure 11 of the original paper) with the predictions of the simple, bisectional model (solid line, with $\psi_1 = \psi_\infty = 0.38$). As one can see, fairly good agreement is obtained.

Next we turn our attention to the data of Chu and Ng concerning pressure drop. These authors plotted in Figure 10 of their paper the ratio of measured (or calculated) permeability to the one of the infinitely extended bed vs. D/d. Taking the reciprocal values of these data (all measurements have been carried out in the D'Arcy regime) the quotient $\Delta P/\Delta P_\infty$ (actual pressure drop to pressure drop of the infinitely extended bed) can be easily obtained. The corresponding results are depicted in Figure 4, using empty circles for measurements are full circles for the output of the computer simulation of Chu and Ng.

These results may not be compared with those of Cohen and Metzner. In fact, the latter authors used ΔP_h (the pressure drop in a homogeneous bed with the average porosity $\bar{\psi}$) and not ΔP_∞ as a scaling factor. Consequently, the comparison carried out by Chu and Ng in Figure 10 of their paper is misleading, and all the conclusions based on it (see Introduction) are irrelevant.

In order to enable comparison, we transformed the $\Delta P/\Delta P_\infty$ quotients to

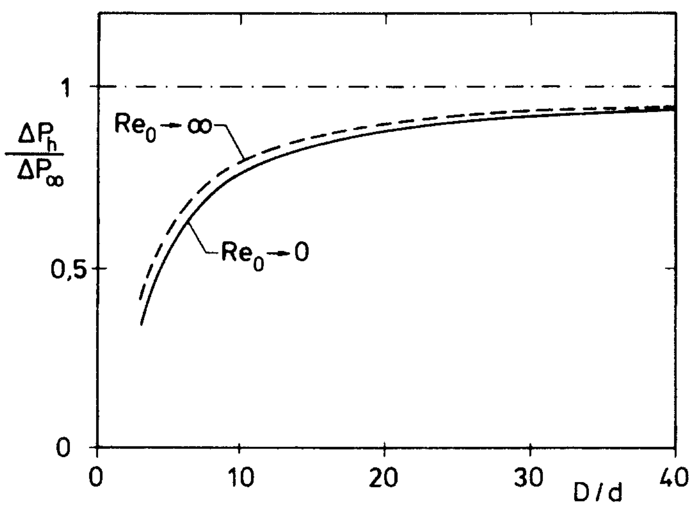

Figure 2. Influence of the change in average porosity with the diameter ratio on pressure drop.

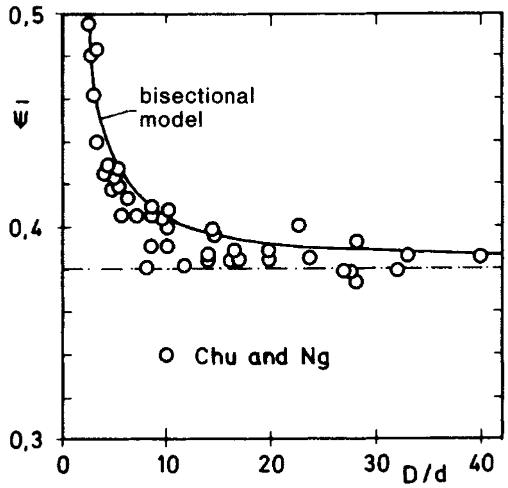

Figure 3. Influence of diameter ratio on average porosity.

$\Delta P/\Delta P_h$ ratios. Doing so, we did not consider every individual point (the correspondence between average porosity and permeability in the plots of Chu and Ng is not always definite). Instead, we approximated the points with a simple exponential function, depicted in Figure 4 as a solid line. With the help of this line and of Eq. 2a (compare Figure 2), we calculated values of the quantity $\Delta P/\Delta P_h$. The latter is plotted in Figure 4 as a broken line.

According to this line, a decrease in the diameter ratio D/d always gives rise to an increase of head loss, a behavior which is the same as the one predicted by Cohen and Metzner.

Actually, the broken line of Figure 4 agrees well with the calculations of Cohen and Metzner and with the experimental data collected by them (see original paper). Consequently, the experimental and theoretical results of Chu and Ng do not contradict those of Cohen and Metzner. On the contrary, they support them.

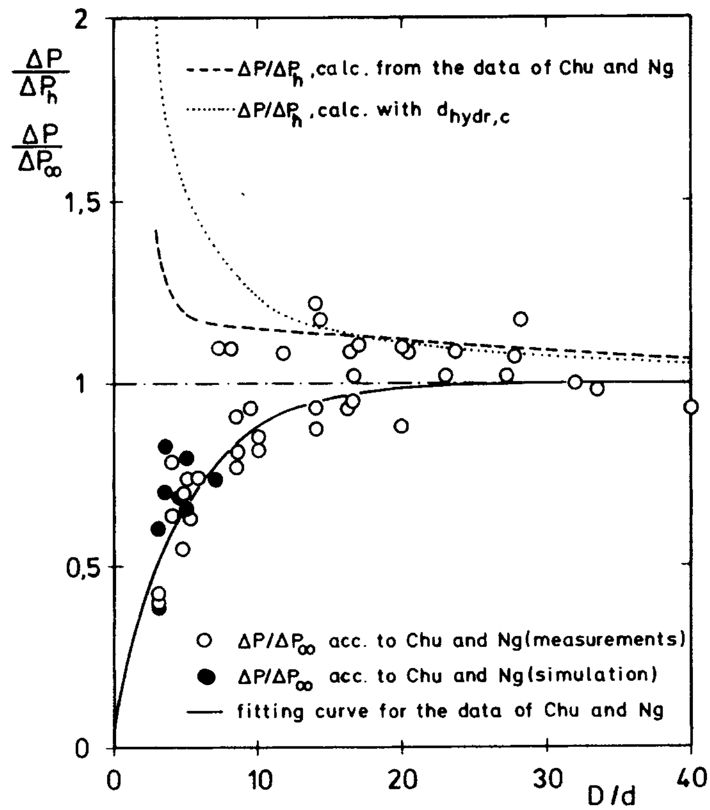

Figure 4. Influence of diameter ratio on pressure drop.

Remarks on the wall friction

As already discussed, maldistribution of porosity and flow in packed tubes is expected to enhance flow and decrease the pressure drop (see Figure 1). However, experimental results (when correctly interpreted) show the opposite behavior (Figure 4). This is obviously due to wall friction, giving rise to increased pressure drop and overcompensating the influence of maldistribution. The impact of wall friction on pressure drop at small Reynolds numbers can be estimated by comparing the solid line of Figure 1 with the broken line of Figure 4. Actually, wall friction must account for the difference between the two curves.

In this context, if should be mentioned that various early workers tried to describe wall friction by accounting for the surface area of the tube in the definition of an hydraulic diameter. For this purpose, they used the corrected hydraulic diameter

$$d_{hydr,c} = d_{hydr}\left[1 + \frac{2}{3}\frac{1}{1-\bar{\psi}}\frac{1}{D/d}\right]^{-1} \quad (3)$$

in the place of the hydraulic diameter of the infinitely extended packing

$$d_{hydr} = \frac{2}{3}\frac{\bar{\psi}}{1-\bar{\psi}}d \quad (4)$$

[See Brauer (1971) and compare with Mehta and Hawley (1969).] Corresponding pressure drop calculations are depicted in Figure 4 as a dotted line. The agreement between this line and the reevaluated data of Chu and Ng (broken curve) is fairly good, at least for $D/d > 10$. However, one should not forget that the deviations of $\Delta P/\Delta P_h$ from unity are not due to wall friction alone, but due to the combination of wall friction and maldistribution. Both effects are accounted for in the model of Cohen and Metzner.

Notation

d = particle diameter, m
d_{hydr} = hydraulic diameter, m
$d_{hydr,c}$ = corrected hydraulic diameter, m
D = tube diameter, m

P = pressure, Pa
Re_0 = Reynolds number, $\bar{u}_0 d/\nu_f$
u_0 = superficial velocity, m · s^{-1}
ΔP = pressure drop, Pa
ν_f = kinematic viscosity of fluid, m^2 · s^{-1}
ρ_f = fluid density, kg · m^{-3}
ψ = porosity
b = according to the bisectional model
h = homogeneous bed with the average porosity $\bar{\psi}$
∞ = infinitely extended bed
1, 2 = core and wall region, respectively
— = average value over the cross-section of the tube

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Reply:

The paper by Chu and Ng (1989) is concerned with flow in a packed tube with a small tube to particle diameter ratio. An interesting feature of this system is that the porosity in such a tube is not uniform. It is more porous in regions next to the walls. To be more exact, the porosity has a limiting value of unity at the wall, and then its magnitude oscillates with decreasing amplitude away from the wall. Because of this porosity profile, the permeability depends on the tube diameter to particle diameter ratio, R $(=D/d_g)$.

As can be seen in Figure 1, our experimental data show that for R larger than 25, the permeability, k, is the same as that of a large tube, k_∞. Between 8 and 25, the permeability can be larger or less than k_∞. Below 8, the permeability is always larger than k_∞. The experimental data of Coulson (1949) and Mehta and Hawley (1969) corroborate our finding that the ratio, k/k_∞, can indeed be less than unity for R in the range between 8 and 25. We have to point out that data reported in the original papers for R larger than 40 are not included in Figure 1. Coulson reports that $k/k_\infty = 0.93$ at $R = 64$, and Mehta and Hawley report that $k/k_\infty = 1.013$ and 0.98 at $R = 91$ for $Re = 1.0$ and 5.0, respectively. All of the three data sets indicate that the permeability ratio decreases with decreasing R in the range of R between 8 and 25, but displays a definite uptrend around $R = 8$.

These data suggest that the presence of the walls has two counteracting effects on fluid flow. A higher porosity promotes flow, but a higher surface area per unit volume hinders it. Let us consider the data in the middle range of R in Figure 1. The permeability ratio can be larger or less than unity. This indicates that the influence of wall friction on fluid flow is close to that due to porosity variation. The reasoning is as follows.

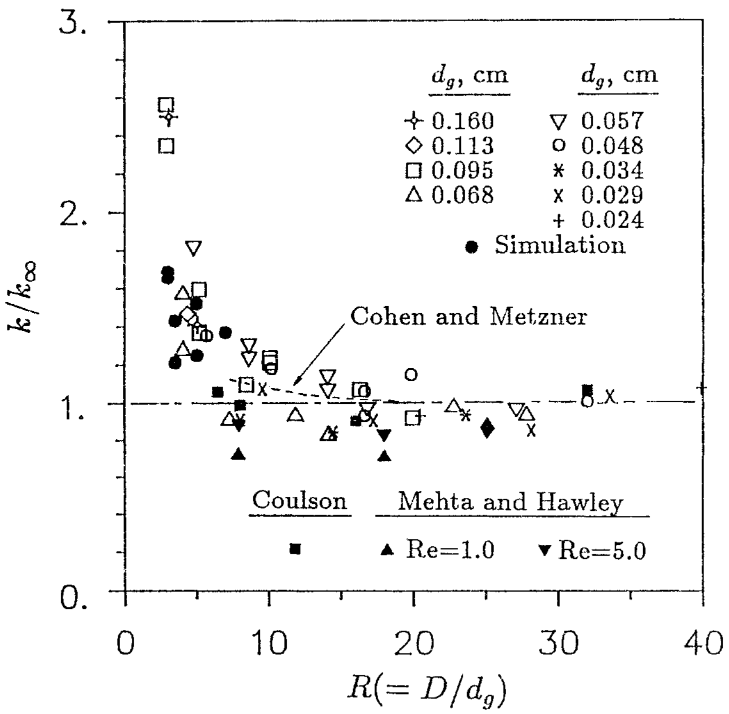

Figure 1. Dependence of permeability on D/d_g.

As reported by Chu and Ng, the overall bed porosity increases with decreasing R. Beginning at around 0.38 for R larger than 30, the porosity increases gradually with decreasing R and then exhibits a more pronounced upward trend for R less than 7 or 8. Since the experimental data of permeability ratio by Chu and Ng, Coulson, and Mehta and Hawley can drop below unity in the range of R between 8 and 25 as R decreases, we conclude that this must be caused by the wall friction. If wall friction is not present, a decrease in R would lead to a higher overall bed porosity and should result in a permeability larger, not smaller, than k_∞. Let us emphasize that this comparison is possible because k_∞ is chosen as the reference permeability.

If R is further reduced to below 8, the permeability ratio is larger than unity because the confining walls cause a marked increase in the overall bed porosity. This indicates that the effect of a higher overall bed porosity completely dominates that of wall friction on pressure drop in this region of R.

Tsotsas and Schlunder correctly point out that the predictions of a parallel capillary tube model (Cohen and Metzner, 1981) actually increase slightly with decreasing R and are in agreement with the experimental data of Chu and Ng (Figure 1). We appreciate it very much. However, it is easy to see that the above discussion is by no means affected by those model predictions as long as some experimental data do decrease with decreasing R for R between 8 and 25. Therefore, we believe that the comments in the paper by

Chu and Ng are indeed relevant and valid.

Let us now turn to the issue of complete distinction between wall friction and porosity variation effects. This is not possible unless an experimentally confirmed model or experimental data for flow in a packed tube with porosity variation, *but without walls*, is available. Without such a model or data, Chu and Ng compared the apparent permeability, k, to the permeability of an infinite porous medium, k_∞. This approach demonstrates the relative importance of the wall friction and porosity variation influences but does not provide a separate quantitative evaluation of each effect.

In order to completely distinguish the two effects on pressure drop, Tsotsas and Schlunder suggest that the experimental results can be compared to the theoretical pressure drop based on a bisectional model, ΔP_b, and that of a homogeneous model, ΔP_h. This is an interesting suggestion but we should keep in mind the following two points.

1. The bisectional model is based on the assumption that the packed tube can be considered to be made up of two regions—a circular core region and an annular region with a thickness equal to half of a particle diameter. Ergun's equation is used for both regions to calculate ΔP_b. This is not a rigorous model for flow in a packed tube without walls. Since the predicted curve in Figure 1 of the letter has not been confirmed, the claim that its deviation from the experimental data of Chu and Ng is caused by wall friction is somewhat tenuous.

2. ΔP_h is also calculated using the Ergun's equation based on the average porosity of the packed tube. The applicability of Ergun's equation for a porous medium with varying porosity is uncertain. Thus, ΔP_h is very much a fictitious quantity, whose physical significance is not completely clear and it is by no means the only meaningful reference pressure drop.

In summary, we feel that this letter provides an interesting perspective to the problem of flow in packed tubes with a small tube to particle diameter ratio. However, because of the uncertainty in both ΔP_b and ΔP_h, the conclusions in this letter, particularly the quantitative ones, should be viewed with a little skepticism.

Notation

A = cross-sectional area
D = tube diameter
d_g = particle diameter
k = absolute permeability
k_∞ = permeability of an infinite porous medium
Q = volumetric flow rate through a porous medium
R = ratio of tube diameter to particle diameter
Re = Reynolds number ($= \rho d_g Q/A\mu$)
ΔP_b = pressure drop based on the bisectional model
ΔP_h = pressure drop based on the homogeneous model
μ = dynamic viscosity
ρ = density

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To the Editor:

In a recent paper titled "The Dead Zone in a Catalyst Particle for Fractional-Order Reactions" (García-Ochoa and Romero, November 1988, p. 1916), we derived an equation (Eq. 11) for the critical value of the Thiele Modulus, that is when the dead zone begins. This equation was developed taking into account the possibility of differences of concentration and temperature between the bulk fluid and the particle surface.

In a previous work titled "Multiplicity Features of Porous Catalytic Pellets: III. Uniqueness Criteria for the Lumped Thermal Model," Hu, Balakotaiah, and Luss (1986) presented a simular equation (Eq. 8) for the same magnitude. This equation does not take into account the particle surface temperature. The authors state that this equation is derived from the work of Joseph and Lundgren (1973) regarding the solution of quasilinear Dirichlet problems driven by positive forces.

In our work (García-Ochoa and Romero, 1988), the equation is deduced following the approach given by Temkin (1982).

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